...keeps you ahead

TEST FORM NUMBER	INSTRUCTIONS TO CANDIDATE	Maximum Marks : 300 Total Questions : 120 Time Allowed : 150 Min.

Read the following instructions carefully before you begin to attempt the questions.

(1) This booklet contains 120 questions.
Mathematics **120 Questions**

(2) All the questions are compulsory.

(3) Before you start to attempt the questions, you must explore this booklet and ensure that it contains all the pages and find that no page is missing or replaced. If you find any flaw in this booklet, you must get it replaced immediately.

(4) **Each question carries negative marking also as 1/3 mark will be deducted for each wrong answer.**

(5) You will be supplied the Answer-sheet separately by the invigilator. You must complete the details of Name, Roll number, Test name/Id and name of the examination on the Answer-Sheet carefully before you actually start attempting the questions. You must also put your signature on the Answer-Sheet at the prescribed place. These instructions must be fully complied with, failing which, your Answer-Sheet will not be evaluated and you will be awarded 'ZERO' mark.

(6) Answer must be shown by completely blackening the corresponding circles on the Answer-Sheet against the relevant question number by **pencil or Black/Blue ball pen** only.

(7) A machine will read the coded information in the OMR Answer-Sheet. In case the information is incompletely/ different from the information given in the application form, the candidature of such candidate will be treated as cancelled.

(8) The Answer-Sheet must be handed over to the Invigilator before you leave the Examination Hall.

(9) Failure to comply with any of the above Instructions will make a candidate liable to such action/penalty as may be deemed fit.

(10) Answer the questions as quickly and as carefully as you can. Some questions may be difficult and others easy. Do not spend too much time on any question.

(11) Mobile phones and wireless communication device are completely banned in the examination halls/rooms. Candidates are advised not to keep mobile phones/any other wireless communication devices with them even switching it off, in their own interest. Failing to comply with this provision will be considered as using unfair means in the examination and action will be taken against them including cancellation of their candidature.

(12) No rough work is to be done on the Answer-Sheet.

(13) No candidate can leave the examination hall before completion of the exam.

NAME OF CANDIDATE:..
DATE :........................ CENTRE CODE :................
ROLL No :..

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

Mathematics

1. If $\vec{a}, \vec{b}$ are two-unit vectors such that $|\vec{a}+\vec{b}| = 2\sqrt{3}$ and $|\vec{a}-\vec{b}| = 6$, then the angle between and $\vec{a}$ and $\vec{b}$, is

(A) $\frac{\pi}{3}$ **(B)** $\frac{2\pi}{3}$
(C) $\frac{\pi}{4}$ **(D)** $\frac{\pi}{2}$

2. For a party 7 guests are invited by a husband and his wife. They sit in a row for dinner. The probability that the husband and his wife sit together, is

(A) $\frac{2}{7}$ **(B)** $\frac{2}{9}$
(C) $\frac{1}{9}$ **(D)** $\frac{4}{9}$

3. In a library 60% of the books are in Hindi, 60% of the remaining books are in English rest of the books are in Malayalam. If there are 4800 books in English, then the total number of books in Malayalam are?

(A) 3400 **(B)** 3500
(C) 3100 **(D)** 3200

Direction (4-6): The general solution of the differential equation $(x^2+x+1)dy + (y^2+y+1)dx = 0$ is $(x+y+1) = A(1+Bx+Cy+Dxy)$where B, Cand D are constants and A is parameter.

4. What is D equal to?

(A) -1 **(B)** 1
(C) -2 **(D)** None of these

5. What is B equal to?

(A) -1 **(B)** 1
(C) 2 **(D)** None of these

6. What is C equal to?

(A) 1 **(B)** -1
(C) 2 **(D)** None of these

7. For the next two, (02) items that follow:
Consider the following data

x_i	f_i
50	2
51	1
52	12
53	29
54	25
55	12
56	10
57	4
58	5

Find the mean of the above data

(A) 70 **(B)** 56.5
(C) 60 **(D)** 54

8. If $\sin^4 a - \cos^4 a = -\frac{2}{3}$, then find the value of $2\cos^2 a$?

(A) $\frac{2}{3}$ **(B)** $\frac{1}{3}$
(C) $1\frac{2}{3}$ **(D)** None of these

9. The length of the perpendicular drawn from (1,2,3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

(A) 4 **(B)** 5
(C) 6 **(D)** 7

10. The point at which the tangent to the curve $= 2x^2 - x + 1$ is parallel to $y = 3x + 9$ is:

(A) (1,2) **(B)** (2,1)
(C) (-2,1) **(D)** (3,9)

11. The equation of a sphere with A(2,3,5) and B(4,9,-3) as the ends of diameter:

(A) $x^2+y^2+z^2-8x-12y+2z=30$
(B) $x^2+y^2+z^2-6x-12y-2z=-20$
(C) $x^2+y^2+z^2+6x-12y+2z=15$
(D) $x^2+y^2+z^2-6x-12y-z=20$

12. If the mean of the square of first n natural numbers is 105, then the median of first n natural numbers is

(A) 8 **(B)** 9
(C) 10 **(D)** 11

13. Consider the following statements:
1. The function f(x) = 3√x is continuous at all except at x = 0.
2. The function f(x) = |x| is continuous at x = 2.99 were [.] is the bracket function.
Which of the above statements is/are correct?

(A) 1 only **(B)** 2 only
(C) Both 1 and 2 **(D)** Neither 1 nor 2

14. Consider the following statements in respect of the differential equation $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$
1. The degree of the differential equation is not defined.
2. The order of the differential equation is 2.
Which of the above statements is/are correct?

(A) 1 only **(B)** 2 only
(C) Both 1 and 2 **(D)** Neither 1 nor 2

15. Sum of the twice the age of Surya and his Father age is 79. Sum of the twice the age of Father and Surya's age is 104. The average of Surya, his Father and his Mother is 32. Then what is the age of his Mother?

(A) 32 **(B)** 33
(C) 34 **(D)** 35

16. If a line makes angle α, β and γ with the coordinate axes respectively, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = ?$

(A) 2 **(B)** –1
(C) 1 **(D)** 2

17. Let the pairs $\overrightarrow{ab}$ and $\overrightarrow{cd}$each determine a plane. Then the planes are parallel if

(A) $(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{d}) = \vec{0}$ **(B)** $(\vec{a}\times\vec{c})\cdot(\vec{b}\times\vec{d}) = \vec{0}$
(C) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$ **(D)** $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \vec{0}$

18. If n(A U B) = 1000, n(A-B) = 100 & n(B-A) = 500 then what is the value of ?

(A) 400 **(B)** 200
(C) 600 **(D)** 800

19. Find $\int x\tan^{-1} x dx$.

(A) $\left(\frac{1}{2}\right)\left(x^2\tan^{-1} x + (x - \tan^{-1} x)\right)$
(B) $\left(\frac{1}{2}\right)\left(x^2\tan^{-1} x - (x - \tan^{-1} x)\right)$
(C) $\left(\frac{1}{2}\right)\left(x^2\tan^{-1} x + x(1 - \tan^{-1} x)\right)$
(D) $x^2\tan^{-1} x + x(1 - \tan^{-1} x)$

20. z is a complex number satisfying, $z^4+z^3+2z^2+z+1=0$, then $|z|$ is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{3}{4}$
(C) 1 **(D)** none of these

21. Which of the following function is not differentiable at x = 1?

(A) f(x) = (x² – 1) |(x – 1) (x – 2)|
(B) f(x) = sin (|x – 1|) - |x – 1|
(C) f(x) = tan (|x – 1|) + |x – 1|
(D) None of these

22. If $P: 4$ is an even prime number,
$q: 6$ is a divisor of 12 and
r: the HCF of 4 and 6 is 2
Which of the following is true?
(A) $(p \wedge q)$ **(B)** $(p \wedge q) \wedge -r$
(C) $-(q \wedge r) \vee p$ **(D)** $-p \vee (q \wedge r)$

23. If z = f of (x) = x^2 where f(x) = x^2, then what is dz/dx equal to?
(A) x^3 **(B)** $2x^3$
(C) $4x^3$ **(D)** $4x^2$

24. What is P(Z = 10) equal to?
(A) 0 **(B)** 1/2
(C) 1/3 **(D)** 1/5

25. What is P(Z is the product of two prime numbers) equal to?
(A) 0 **(B)** 1/2
(C) 1/4 **(D)** None of these

26. If $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$ then, evaluate $\sum_{n=1}^{N} U_n$
(A) 1 **(B)** 0
(C) 2 **(D)** None of these

27. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is
(A) $\frac{4}{3}$ **(B)** $\frac{4}{\sqrt{3}}$
(C) $\frac{2}{\sqrt{3}}$ **(D)** None of these

28. A straight line through point $A(3,4)$ is such that its intercept between the axes is bisected at A. its equation is
(A) $x + y = 7$ **(B)** $3x - 4y + 7 = 0$
(C) $4x + 3y = 24$ **(D)** $3x + 4y = 25$

29. The latus rectum of the hyperbola $9x^2 - 16y^2 + 72x - 32y - 16 = 0$ is:
(A) 9/2 **(B)** -9/2
(C) 32/3 **(D)** -32/3

30. If $5a = \sec\alpha$ and $\frac{5}{a}$ then find the value of $25\left(a^2 - \frac{1}{a^2}\right)$.
(A) 2 **(B)** -1
(C) 1 **(D)** 0

31. If $\int \frac{\sqrt{1-\alpha^2}}{\alpha^4} dx = f(\alpha)\left[\sqrt{1-\alpha^2}\right]^n + k$, where k constant of integration, then $[f[\alpha]]^n$ =?
(A) $\frac{-1}{27\alpha^6}$ **(B)** $\frac{1}{9\alpha^4}$
(C) $\frac{-1}{3\alpha^3}$ **(D)** $\frac{-1}{27\alpha^3}$

32. The maximum value of the function $f(x) = x^3 + 2x^2 - 4x + 6$ exists at:
(A) -2 **(B)** -3/2
(C) 2 **(D)** 3/2

33. Consider the function $f(x) = \frac{x^2-1}{x^2+1}$, where $x \in R$
What is the minimum value of $f(x)$?
(A) -3 **(B)** -1
(C) 1 **(D)** 3

34. Let N denote the set of all non-negative integers and Z denote the set of all integers. The function f : Z→N given by f(x)=|x| is:
(A) One-one but not onto **(B)** Onto but not one-one
(C) Both one-one and onto **(D)** Neither one-one onto

35. Sum of series in AP having 14 terms is 875. If first term is 17, then find a14
(A) 105 **(B)** 103
(C) 108 **(D)** 104

36. Let $f(x) = \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}, x = 4$. The value which should be assigned to $f(x)$ at $x = \frac{\pi}{4}$ at, so that it is continuous there at, is
(A) 1 **(B)** 1/2
(C) 2 **(D)** none of these

37. The distance between the points A (0, 6) and B (0, –2) is -
(A) 8 **(B)** 10
(C) 4 **(D)** 9

38. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then
(A) *a/b* is one of the cube roots of unit
(B) *a* is one of the cube roots of unity
(C) *b* is one of the cube roots of unity
(D) *a/b* is one of the cube roots of -1

39. Find whether the function $f(x) = x^4 + 3x^2 + 7$, is-
(A) Odd function **(B)** Even function
(C) Neither odd or even **(D)** Both

40. If letters of the word 'KUBER' are written in all possible orders and arranged as in a dictionary, then rank of the word 'KUBER' will be:
(A) 67 **(B)** 47
(C) 57 **(D)** 37

41. Find $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$
(A) $e.3^{-3x} + k$ **(B)** $\frac{x^3}{3} + k$
(C) $e^{3\text{te } x} + k$ **(D)** None of the above

42. What is the value of 34% of 11% of 14
(A) 0.3426 **(B)** 0.6236
(C) 0.4576 **(D)** 0.5236

43. Which of the following is greatest?
$\frac{42}{23}, \frac{31}{20}, \frac{57}{36}, \frac{82}{51}$
(A) $\frac{57}{36}$ **(B)** $\frac{31}{20}$
(C) $\frac{42}{23}$ **(D)** $\frac{82}{51}$

44. The mean and standard deviation of 100 items are 50, 5 and that of 150 items are 40, 6 respectively. What is the combined standard deviation of all 250 items?
(A) 7.1 **(B)** 7.3
(C) 7.5 **(D)** 7.7

45. Let $f(x)$ be a function defined in $1 \leq x < \infty$ by $f(x) = \begin{cases} 2 - x & \text{for } 1 \leq x \leq 2 \\ 3x - x^2 & \text{for } x > 2 \end{cases}$
What is the differentiable coefficient of f(x) at x = 3 ?
(A) 1 **(B)** 2
(C) -1 **(D)** -3

46. Find $\int \frac{3}{x^2+5x+6} dx$.
(A) $3\tan^{-1}(x+2) + c$ **(B)** $2\tan^{-1}(x+3) + c$
(C) $3\ln\left(\frac{x+2}{x+3}\right) + c$ **(D)** $-3\ln\left(\frac{x+2}{x+3}\right) + c$

47. The function $f(x) = \sin^4 x + \cos^4 x$ increases if :

(A) $0 < x < \frac{\pi}{8}$ **(B)** $\frac{\pi}{4} < x < \frac{3\pi}{8}$
(C) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ **(D)** $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

48. When the Discriminant (b^2-4ac) is negative there are:
(A) 2 non-real solutions
(B) 2 real and unequal solutions
(C) 1 real solution
(D) none of these

49. What will be $\tan x + \tan 2x + \tan x \cdot \tan 2x = ?$; (if $x = 15°$);
(A) 0 **(B)** 1
(C) $\frac{1}{\sqrt{3}}$ **(D)** $\sqrt{3}$

50. A flower-bed in the form of a sector has been fenced by a wire of 40 m length. If the flower-bed has the greatest possible area, then what is the radius of the sector?
(A) 25 m **(B)** 20 m
(C) 10 m **(D)** 5 m

51. Find the eccentricity of an ellipse whose latus rectum is one half of its major axis.
(A) $\frac{3}{3}$ **(B)** $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{1}}{2}$ **(D)** $\frac{3}{2}$

52. The locus of the midpoint of a chord of the circle $x^2 + y^2 - 2x - 2y - 2 = 0$ which makes an angle of 120° at the centre is:
(A) $x^2 + y^2 - 2x - 2y + 1 = 0$
(B) $x^2 + y^2 - x + y - 1 = 0$
(C) $x^2 + y^2 - 2x - 2y - 1 = 0$
(D) None of these

53. In a charity show tickets numbered consecutively from 101 through 350 are placed in a box. What is the probability that a ticket selected at random (blindly) will have a number with the hundredth digit of 2?
(A) 050 **(B)** 0.40
(C) 0.10 **(D)** 0.57

54. Two numbers are selected randomly from the set S = {1,2,3,4,5,6} without replacement one by one. The probability that minimum of the two number is less than 4, is
(A) 1/15 **(B)** 3/5
(C) 4/5 **(D)** 14/15

55. In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of 5th and 6th terms is zero then a/b equal to
(A) $\frac{5}{n-4}$ **(B)** $\frac{6}{n-5}$
(C) $\frac{n-4}{5}$ **(D)** None of these

56. Let $AX = B$ represents a system of equations where A is 2×3 real matrix. The system is known to be inconsistent. The highest possible rank of A is
(A) 1 **(B)** 2
(C) 3 **(D)** Can't be determined

57. Solution of the differential equation $(y + x\sqrt{xy}(x + y))dx + (y\sqrt{xy}(x + y) - x)dy = 0$ is
(A) $\frac{x^2+y^2}{2} + \tan^{-1}\sqrt{\frac{y}{x}} = c$
(B) $\frac{x^2+y^2}{2} + 2\tan^{-1}\sqrt{\frac{x}{y}} = c$
(C) $\frac{x^2+y^2}{2} + 2\cot^{-1}\sqrt{\frac{x}{y}} = c$
(D) None of these

58. Find the general solution of equation: -
$\cot\frac{x}{2} - \operatorname{cosec}\frac{x}{2} = \cot x$
(A) $x = n\pi \pm \frac{2\pi}{3}$ **(B)** $x = 2n\pi \pm \frac{2\pi}{3}$
(C) $x = 4n\pi \pm \frac{2\pi}{3}$ **(D)** None

59. The equation $y^2 - x^2 + 2x - 1 = 0$ represents
(A) a hyperbola
(B) an ellipse
(C) a pair of straight lines
(D) a rectangular hyperbola

60. If e is the eccentricity of ellipse $= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where a < b), then
(A) $b^2 = a^2 (1 - e^2)$ **(B)** $a^2 = b^2 (e^2 - 1)$
(C) $a^2 = b^2 (1 - e^2)$ **(D)** $b^2 = a^2 (e^2 - 1)$

61. Find the integration of function $f(x) = \frac{x^2+1}{x^2-1}$ with respect to :
(A) $x + \log\left(\frac{x-1}{x+1}\right) + c$ **(B)** $x + \tan^{-1} x + c$
(C) $x + \log\frac{1+x}{1-x} + c$ **(D)** NOT

62. In an examination it is required to get 1034 of the aggregate marks to pass. A student gets 940 marks and is declared failed by 5% marks. What are the maximum aggregate marks a student can get?
(A) 1620 **(B)** 1880
(C) 1750 **(D)** None of these

63. Consider the following statements:
1. The function f(x) = sin x decreases on the interval (0, π/2).
2. The function f(x)= cos x increases on the interval (0, π/2).
Which of the above statements is/are correct?
(A) 1 only **(B)** 2 only
(C) Both 1 and 2 **(D)** Neither 1 nor 2

64. Suresh age is 125% of what it was ten years ago, but 250/3% of what it will be after ten years. What is the present age of Suresh?
(A) 60 years **(B)** 50 years
(C) 40 years **(D)** Cannot be determined

65. The point $P(a, b)$ lies on the straight line $3x + 2y = 13$ and the point $Q(b, a)$ lies on the straight line $4x - y = 5$, then the equation of the line PQ is
(A) $x - y = 5$ **(B)** $x + y = 5$
(C) $x + y = -5$ **(D)** $x - y = -5$

66. The mean of 6 observations is 8 and that of 4 observations is 10. What is the mean of all the 10 observations?
(A) 7.5 **(B)** 5.6
(C) 8.8 **(D)** 9.2

67. Let $f(t) = \frac{|t|\left(3e^{\frac{1}{k}}+4\right)}{2-e^{\frac{1}{|t|}}}, t \neq 0$ and $f(0) = 0$ then
(A) f is not continuous
(B) f is continuous but not differentiable at t = 0
(C) $f'(0)$ exist
(D) $f'(0) + = 2$

68. The equation of the plane whose intercepts on the coordinate axes are $-4, 2$ and 3 is
(A) $3x + 6y + 4z = 12$ **(B)** $-3x + 6y + 4z = 12$
(C) $-3x - 6y - 4z = 12$ **(D)** none of these

69. If $|z| = k$ and $\omega = \frac{z-k}{z+k}$, then $\text{Re}(\omega) =$

(A) 0 **(B)** k
(C) $\frac{1}{k}$ **(D)** $-\frac{1}{k}$

70. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the all three of them are divisible by both 2 and 3 is
The ways of selecting three distinct numbers are chosen randomly from the first 100 natural numbers is
(A) $\frac{4}{25}$ **(B)** $\frac{4}{35}$
(C) $\frac{4}{33}$ **(D)** $\frac{4}{1155}$

71. Consider the following relations from A to B where A = {u, v, w, x, y, z} and B = {p, q, r, s}.
1. {(u, p), (v, p), (w, p), (x, q), (y, q), (z, q)}
2. {(u, p), (v, q), (w, r), (z, s)}
3. {(u, s), (v, r), (w, q), (u, p), (v, q), (z, q),}
4. {(u, q), (v, p), (w, s), (x, r), (y, q), (z, s),}
Which of the above relations are not functions?
(A) 1 and 2 **(B)** 1 and 4
(C) 2 and 3 **(D)** 3 and 4

72. The compound interest on a certain sum of money at a certain rate per annum for two years is Rs. 4,100 and the simple interest on the same amount of money at the same rate for 3 years is Rs. 6000. Then the sum of money is :
(A) Rs. 40,000 **(B)** Rs. 36,000
(C) Rs. 42,000 **(D)** Rs. 50,000

73. Find the middle term of the following series 3, 6, 9, 12………………,198
(A) 105 **(B)** 99
(C) 96 **(D)** 102

74. What will be $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=?$
(A) 1 **(B)** $2\tan\,\theta$
(C) $2\tan\,2\theta$ **(D)** $\tan\,2\theta$

75. If $f(4)=4, f'(4)=1$, then $\lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}$, then is equal to
(A) 1 **(B)** -1
(C) 2 **(D)** -2

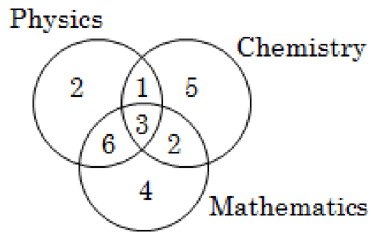
76. If A = {2, 7} and B = {2, 3}, then number of ordered pairs in B × B.
(A) 9 **(B)** 7
(C) 4 **(D)** 8

77. If *a, 4, b* are in AP and *4, a, b* are in GP, then find *a* and *b*. [given *a* + *b*]
(A) -6,12 **(B)** 2,6
(C) -4,4 **(D)** -8,16

78. If the equation $2x^2+7xy+3y^2-9x-7y+k=0$ represents a pair of lines, then k is equal to
(A) 4 **(B)** 2
(C) 1 **(D)** -4

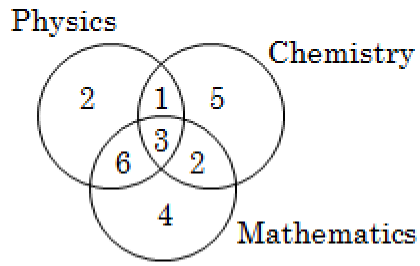
Direction (79-81): In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects.

79. Consider the following statements:
1. The number of students who had taken only one subject is equal to the number of students who had taken only two subject.
2. The number of students who had taken at least two subjects is four times the number of students who had taken all the three subjects.
Which of the above statements is/are correct?



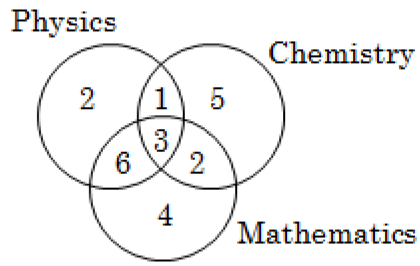
(A) 1 only **(B)** 2 only
(C) Both 1 and 2 **(D)** Neither 1 nor 2

80. The number of students who had taken only two subjects is:



(A) 7 **(B)** 8
(C) 9 **(D)** 10

81. The number of students who had taken only physics is:



(A) 2 **(B)** 3
(C) 5 **(D)** 6

82. If log a, log b, log c and (log a – log 2b), (log 2b – log3c), (log 3c – log a) are in arithmetic progression then
(A) 18 (a + b + c)^2 = 18(a^2 + b^2 +c^2) + ab
(B) a, b, c are in AP
(C) a, 2b, 2c are in HP
(D) a, b, c can be the lengths of the sides of a triangle. (Assume all logarithmic terms to be defined)

83. Find the mean deviation about the median for the data. 6, 7, 4, 8, 9, 10, 3, 5
(A) 2 **(B)** 3
(C) 4 **(D)** 5

84. Let $f: R\to R$ be a function satisfying
$f(x+y)=f(x)+\lambda xy+3x^2y^2$ for all $x, y\in R$.
if $f(3)=4$ and $f(5)=52$, then $f'(x)$ is equal to
(A) $10x$ **(B)** $-10x$
(C) $20x$ **(D)** $128x$

85. Find $\int\frac{4}{\sqrt{-x^2+4x}}dx$.
(A) $4\sin^{-1}\frac{(x-2)}{2}+c$ **(B)** $2\sin^{-1}\frac{(x-2)}{2}+c$
(C) $4\tan^{-1}\frac{(x-2)}{2}+c$ **(D)** $-2\sin^{-1}\frac{(x-2)}{2}+c$

86. a, b, c and u, v, w are the vertices of two triangles such that $c = (1-r)a + rb$ and $\omega = (1-r)v + ru$ where r is a complex number, then the two triangles
(A) Have the same area **(B)** Are similar
(C) Are congruent **(D)** None of these

87. Consider the function $f(x) = \frac{1-\sin x}{(\pi-2x)^2}$ Where $x \neq \frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = \lambda$. What is $\lim_{x\to\frac{\pi}{2}} f(x)$ equal to?
(A) 1 **(B)** 1/2
(C) 1/4 **(D)** 1/8

88. If $f(x) = xe^{x(1-x)}$, then $f(x)$ is
(A) Increasing on $[-1/2, 1]$
(B) Decreasing on $[-\infty, -1/2] \cup [1, \infty)$
(C) A and B both
(D) none of these

89. If $f(x) = \sin x - \cos x$ the function decreasing in $0 \leq x \leq 2\pi$ is:
(A) $\left[\frac{5\pi}{6}, \frac{3\pi}{4}\right]$ **(B)** $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
(C) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ **(D)** None of these

90. What is the value of tan450°
(A) 1 **(B)** 1/4
(C) 1/2 **(D)** not defined

91. In expansion of $\left[2^{\log_2 \sqrt{9x-1+7}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$, 6 th term is 84 . Then $x =$
(A) 2 **(B)** 1
(C) 1 or 2 **(D)** 3 or 4

92. If $n = \frac{\pi}{4\alpha}$, then $\tan\alpha \cdot \tan(2n-1)\alpha$ is equal to
(A) 1 **(B)** -1
(C) ∞ **(D)** None of these

93. Let, $f(x) = \log_e |x-1|$ then the value of $f'\left(\frac{1}{2}\right)$ is
(A) -2 **(B)** 2
(C) non-existent **(D)** 1

94. What is $\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A}$ equal to?
(A) sec A – tan A **(B)** 2 sec A. tan A
(C) 4 sec A. tan A **(D)** 4 cosec A. cot A

95. What is the equation of the line through (1, 2) so that the segment of the line intercepted between the axes is bisected at this point?
(A) 2x – y = 4 **(B)** 2x – y + 4 = 0
(C) 2x + y = 4 **(D)** 2x + y + 4 = 0

96. The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is
(A) $-\frac{1}{xy} = C$ **(B)** $-\frac{1}{xy} + \log y = C$
(C) $\frac{1}{xy} + \log y = C$ **(D)** $\log y = Cx$

97. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$ where k is a constant of integration, then A+B+C equals:
(A) $\frac{16}{5}$ **(B)** $\frac{8}{5}$
(C) $\frac{11}{5}$ **(D)** $\frac{7}{5}$

98. If $\int y^5 e^{-y^2} dy = g(y)e^{-y^2} + k$ then $g(+1) = ?$
(A) -1 **(B)** 1
(C) $\frac{-5}{2}$ **(D)** $\frac{-1}{2}$

99. Let a, b, c R such that no two of them are equal and satisfy $\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix} = 0$, then equation $24ax^2 + 4bx + c = 0$ has
(A) At least one root in [0, 1]
(B) At least one root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(C) At least one root in [–1, 0]
(D) At least two roots in [0, 2]

100. If m and M respectively the minimum and maximum of $f(x) = (x-1)^2 +$ for $x \in [-3, 1]$, then the ordered pair is equal to
(A) $(-3, 19)$ **(B)** $(3, 19)$
(C) $(-19, 3)$ **(D)** $(-19, -3)$

101. For what value of x, the fifth term of the following expansion is equal to 105?
$\left(\frac{1}{2\sqrt{x}} - \frac{1}{2}\right)^{10}$
(A) 1/2 **(B)** 1/4
(C) 1/6 **(D)** 1/8

102. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$ where n is a positive integer, the sum of the coefficients of x^5 and x^{10} is 0. What is n equal to?
(A) 5 **(B)** 10
(C) 15 **(D)** None of these

103. How many 3 digit number can be formed with the digits 5, 6, 2, 3, 7 and 9 which are divisible by 5 and none of its digit is repeated?
(A) 12 **(B)** 16
(C) 20 **(D)** 24

104. A seller calculated his intended selling price at 6% profit on the cost of a product. However, owing to some mistake while selling, the units and tens digits of the selling price got interchanged. This reduced the profit by Rs. 180 and profit percentage to 2.4%. What is the cost price of the product?
(A) Rs. 4500 **(B)** Rs. 5000
(C) Rs. 4750 **(D)** Rs. 6000

105. $5x^2 + 2x + 1 = 0$ and find the nature of roots
(A) non-real **(B)** real and equal
(C) real and unequal **(D)** none of the above

106. If in a triangle, R and r are the circumradius and inradius respectively, then the HM of the of the triangle is
(A) 3r **(B)** 2r
(C) R+r **(D)** None of these

107. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\{1 - (\sin x - \cos x)^2\}} dx$
(A) $\frac{1}{20}(\log 3 - \log 2)$ **(B)** $\frac{1}{10}(\log 3)$
(C) $\frac{1}{20}(\log 3)$ **(D)** $\frac{1}{20}(\log \sqrt{3})$

108. If $f(x) = \frac{3x+2}{5x-3}$, then
(A) $f^{-1}(x) = f(x)$ **(B)** $f^{-1}(x) = -f(x)$
(C) $f^{-1}(x) = -f(x)$ **(D)** $f^{-1}(x) = -f(x)$

109. Find the general solution of equation: - $\sin 9\theta = \sin \theta$
(A) $\theta = \frac{n\pi}{10}, \frac{n\pi}{4}$ **(B)** $\theta = \frac{9n\pi}{10}, \frac{3n\pi}{4}$
(C) $\theta = \frac{n\pi}{4}, \frac{(2n+1)\pi}{4}$ **(D)** None

110. A company wishes to gain 25% after allowing 10% discount on the market price to his Customers. At what approximate percent higher than the cost price must they mark their goods?

(A) 19% **(B)** 39%
(C) 27% **(D)** 42%

111. The sum of the series formed by the sequence 3, √3, 1 …….. upto infinity is:

(A) $\frac{3\sqrt{3(\sqrt{3}+1)}}{2}$ **(B)** $\frac{3\sqrt{3}(\sqrt{3}-1)}{2}$
(C) $\frac{3(\sqrt{3}+1)}{2}$ **(D)** $\frac{3(\sqrt{3}-1)}{2}$

112. Mr. X has three sons namely P, Q and R. P is the eldest son of Mr. X while R is the youngest one. The present ages of all three of them are square numbers. The sum of their ages after 5 years is 44. What is the age of P after three years?

(A) 1 **(B)** 19
(C) 9 **(D)** 16

113. A man borrows Rs. 8000 at 20% compound rate of interest. At the end of each year he pays back Rs. 3000. How much amount should he pay at the end of the third year to clear all his dues?

(A) Rs. 5492 **(B)** Rs. 5552
(C) Rs. 5904 **(D)** Rs. 6933

114. If A = {1, 3, 5} and B = {2, 3}, then number of ordered pairs in

(A) 8 **(B)** 6
(C) 9 **(D)** 5

115. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three-unit vectors inclined to each other at an angle θ, then the maximum value of θ is

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{2}$
(C) $\frac{2\pi}{3}$ **(D)** $\frac{5\pi}{6}$

116. Find the general solution of equation: - $3\tan^2\theta - 2\sin\theta = 0$

(A) $\theta = n\pi$ or $n\pi \pm \frac{\pi}{6}$ **(B)** $\theta = n\pi$ or $n\pi + (-1)^n \frac{\pi}{6}$
(C) $\theta = n\pi$ or $2n\pi \pm \frac{\pi}{6}$ **(D)** None

117. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vector, then the vector which is equally inclined to these vectors is

(A) $\vec{a} + \vec{b} + \vec{c}$ **(B)** $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$
(C) $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$ **(D)** $|\vec{a}|\,\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\,\vec{c}$

118. If $\frac{1}{2}, \frac{1}{3}, n$ are direction cosines of a line, then the value of *n* is

(A) $\frac{\sqrt{23}}{6}$ **(B)** $\frac{23}{6}$
(C) $\frac{2}{3}$ **(D)** $\frac{1}{6}$

119. $\lim_{x\to 0} \frac{(1+x)^n - 1}{x}$ is equal to:

(A) 0 **(B)** 1
(C) n **(D)** n -1

120. Consider the curve y = e^{2x}. Where does the tangent to the curve at (0, 1) meet the x-axis.

(A) (1, 0) **(B)** (2, 0)
(C) (–1/2, 0) **(D)** (1/2, 0)

Mathematics

1. **Answer: (B)**

Let θ be the angle between unit vectors $\vec{a}$ and $\vec{b}$ ..

Since $\vec{a}$ and $\vec{b}$. are unit vectors.

$\therefore \tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$

$\Rightarrow \tan\frac{\theta}{2} = \frac{6}{2\cdot\sqrt{3}} = \sqrt{3}$

$\Rightarrow \frac{\theta}{2} = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3}$

2. **Answer: (B)**

We have,

7 guests and one couple in a party.

So, the total number of ways in which 7 guests and the couple can sit in a row is 9!

Number of ways in which the husband and his wife sit together is 2! × 8!

Required probability = $\frac{2!\times 8!}{9!} = \frac{2}{9}$

3. **Answer: (D)**

Let there are X books in the library.

Hindi books = 60% of X = 60 ×/100 = 0.6X

Remaining Books $= X - 0.6X = 0.4X$

English books = 40% of reaming books = 60% of 0.4X = 0.24X

Malayalam Books $= X - 0.6X - 0.24X = 0.16X$

Given, $0.24X = 4800$

$X = \frac{4800}{0.24} = 20000$

Malayalam Books = 0.16X = 0.16*20000 = 3200.

Answer: (4-6)

$(x^2+x+1)dy = -(y^2+y+1)dx$

$\frac{dx}{(1+x+x^2)} + \frac{dy}{(1+y+y^2)} = 0$

$\Rightarrow \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 0$

$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)$

$= \frac{2}{\sqrt{3}}\tan^{-1} C_1$

$\Rightarrow \tan^{-1}\left\{\frac{\left(\frac{2x+2}{\sqrt{3}}\right)+\left(\frac{2y+1}{\sqrt{3}}\right)}{1-\left(\frac{2x+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right\} = \tan^{-1} C_1$

$\left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$\Rightarrow \frac{\sqrt{3}[(2x+1)+2y+1)]}{3-(2x+1)(2y+1)} = C_1$

$\Rightarrow \frac{2\sqrt{3}(x+y+1)}{-4xy-2y-2x+2} = C_1$

$\Rightarrow 2\sqrt{3}(x+y+1) = C_1(2-2x-2y-4xy)$

$\Rightarrow 2\sqrt{3}(x+y+1) = 2C_1(1-x-y-2xy)$

$\Rightarrow 2\sqrt{3}(x+y+1) = 2C_1(1-x-y-2xy)$

$(x+y+1) = \frac{c_1}{\sqrt{3}}(1-x-y-2xy)$

$(x+y+1) = A(1+Bx+Cy+Dxy)$

4. **Answer: (C)**

$D = -2$

5. **Answer: (A)**

$B = -1$

6. **Answer: (B)**

$C = -1$

7. **Answer: (D)**

Let A = 54

x_i	f_i	$d_i = x_i - A$	d_i^2	$f_i d_i$	$f_i d_i^2$
50	2	-4	16	-8	32
51	1	-3	9	-3	9
52	12	-2	4	-24	48
53	29	-1	1	-29	29
54	25	0	0	0	0
55	12	1	1	12	12
56	10	2	4	20	40
57	4	3	9	12	36
58	5	4	16	20	80
Total	100			0	286

Mean = A + $\frac{\sum f_i d_i}{N}$ = 54+0 = 54

8. **Answer: (C)**

9. **Answer: (D)**

Let P be the point (1, 2, 3) and PN be the length of the perpendicular from P on the given line. Coordinates of point N are .

Now PN is perpendicular to the given line 3i+2j-2k

$3(3\lambda+6-1)+2(2\lambda+7-2)-2(-2\lambda+7-3)=0$

$\lambda = -1$

Then, point N is (3, 5, 9)

PN = 7

10. **Answer: (A)**

The equation of the curve

$y = 2x^2 - x + 1$

The slope of the tangent, $\frac{dy}{dx} = 4x - 1$ ...i

This is parallel to the line $y = 3x + 9$...ii

Slope of the line $m = 3$

So, the slope of both eqn will be equal

$4x - 1 = 3 \Rightarrow x = 1$

So, $y = 2(1)^2 - 1 + 1 = 2$, hence the point (1,2)

11. **Answer: (B)**

The points of the diagonal are A (2,3,5) and B (4,9,-3)

Thus, the equation of the circle is

(x-2) (x-4) + (y-3) (y-9) + (z-5) (z+3) = 0

$x^2 - 6x + 8 + y^2 - 12y + 27 + z^2 - 2z - 15 = 0$

$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$

12. **Answer: (B)**

We know that,

$x_1, x_2, \ldots \ldots x_n$ are n values of a variable X, then mean $\bar{X} = \frac{x_1+x_2+x_3+\cdots x_n}{n}$

We have,

$\frac{1^2+2^2+\cdots\ldots+n^2}{n} = 105$

We know that,

$1^2+2^2+\cdots\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow \frac{(n+1)(2n+1)}{6} = 105$

$\Rightarrow 2n^2 + 3n - 629 = 0$

$\Rightarrow (2n+37)(n-17) = 0$

$\Rightarrow n = 17$

Thus given numbers are 1,2,3,4, ... ,16,17.

We know that median is the middle value of a distribution.

So, 9 is their Median.

13. **Answer: (B)**

1.f $f(x) = \sqrt[3]{x}$

$\lim_{x\to 0^+} \sqrt[3]{x} = \lim_{x\to 0^+} f(x) = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt[3]{x} = 0$

$f(0) = 0$

LHL $= f(0) = \text{RHL}$

∴ continuous at 0 as well

2. $f(x) = [x]$

$\lim_{x\to 2.99^+} [x] = 2[x \to 2.99^+; x = 2.9900 \ldots\ldots; \Rightarrow [x] = 2]$

$x \to 2.99^+; x = 2.9900 \ldots 1; \Rightarrow [x] = 2$

$\lim_{x\to 2.99^-} [x] = \lim_{x\to 2.99^-} 2 = 2$

$f(2{,}99) = [2{,}99] = 2$

$\text{LHL} = f(0) = \text{RHL}$

⇒ It is continuous at 2.99

14. **Answer: (C)**

Statement I: Differential equation is not a polynomial equation in its derivatives. So, its degree is not defined.

Statement II: The highest order derivative in the given polynomial is 2.

15. Answer: (D)

$2S + F = 79$
$2F + S = 104$
$S = 18F = 43$
$18 + 43 + M/3 = 32$
$M = 35$

16. Answer: (B)

We have,
$l = \cos\alpha, m = \cos\beta$ and $n = \cos\gamma$
$\therefore l^2 + m^2 + n^2 = 1$
$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$
$\left[\because \cos^2 a = \frac{1+\cos 2a}{2}\right]$
$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

17. Answer: (C)

$\vec{a} \times \vec{b}$ is a vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Similarly, $\vec{c} \times \vec{d}$ is a vector perpendicular to the plane containing $\vec{c}$ and $\vec{d}$.
Thus, the two planes will be parallel if their normal, i.e., $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$, are parallel.
$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

18. Answer: (A)

n (A U B) = n (A-B) + n (B-A) + n$(A \cap B)$ {Venn Diagram results}
1000 = 100 + 500 + n$(A \cap B)$
n$(A \cap B) = 1000 - 600 = 400$

19. Answer: (B)

$\int x\tan^{-1} x dx = \int_2 x\tan_1^{-1} x dx$
Using integration by part,
$\Rightarrow \tan^{-1} x \int x dx - \int \left(\frac{d\tan^{-1} x}{dx} \int x dx\right) dx$
$\Rightarrow \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{1}{x^2+1} \cdot \frac{x^2}{2} dx$
$\Rightarrow \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2+1-1}{x^2+1} \cdot \left(\frac{1}{2}\right) dx$
$\Rightarrow \tan^{-1} x \cdot \frac{x^2}{2} - \int \left(\frac{1}{2}\right)\left(1 - \frac{1}{x^2+1}\right) dx$
$\Rightarrow \left(\frac{1}{2}\right)\left(x^2\tan^{-1} x - (x - \tan^{-1} x)\right)$

20. Answer: (C)

We have,
$z^4 + z^3 + 2z^2 + z + 1 = 0$
$\Rightarrow (z^4 + z^3 + z^2) + (z^2 + z + 1) = 0$
$\Rightarrow z^2(z^2 + z + 1) + (z^2 + z + 1) = 0$
$\Rightarrow (z^2 + z + 1)(z^2 + 1) = 0$
$\Rightarrow z = \pm i, \omega, \omega^2$
$\Rightarrow |z| = 1$

21. Answer: (C)

f(x) = (x² – 1)|(x – 1) (x – 2)|
f(x) = (x² – 1)|(x – 1) (x – 2)|
= (x + 1) [(x – 1)|x – 1|]|x – 2|
Which is differentiable at x = 1
For f(x) = sin (|x – 1|) - |x – 1|
$f'(1^+) = \lim_{h\to 0} \frac{\sin h - h - 0}{h} = 0$
$f'(1^-) = \lim_{h\to 0} \frac{\sin |-h| - |-h|}{-h} = \lim_{h\to 0} \frac{\sin h - h}{-h} = 0$
Hence, f(x) is differentiable at x = 1.
For f(x) = tan (|x – 1|) + |x – 1|
$f'(1^+) = \lim_{h\to 0} \frac{\tan h + h - 0}{h} = 2$
$f'(1^-) = \lim_{h\to 0} \frac{\tan |-h| + |-h|}{-h} = \lim_{h\to 0} \frac{\tanh + h}{-h} = -2.$
Hence, f(x) is non-differentiable at x = 1.

22. Answer: (D)

Clearly p is false and q and r are true. The option (D) is true because it is $-p \vee (q \wedge r)$ which means not p or (q and r) which means p is not correct and q and r are correct.

23. Answer: (C)

Z = fof (x) = f(x²) = x^4
$\frac{dz}{dx} = 4x^3$

24. Answer: (A)

$A = \{1,2,3,4,5,6,7\}$ and $z = x + y$
x = set of odd number
y = set of even number
$P(Z = 10) = \frac{n(E_2)}{n(S)} = \frac{0}{12} = 0$

25. Answer: (C)

$A = \{1,2,3,4,5,6,7\}$ and $z = x + y$
x = set of odd number
y = set of even number
Z = product of two prime numbers Z = x + y = 7 + 6 = 13
$n(E_4) = 3$
$P(Z = 9) = \frac{n(E_4)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

26. Answer: (B)

Given:

The determinant $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$

We have to find the value of $\sum_{n=1}^{N} U_n$ Now, by properties of determinant-7, we get that $\sum_{n=1}^{N} U_n = \begin{vmatrix} \sum_{n=1}^{N} n & 1 & 5 \\ \sum_{n=1}^{N} n^2 & 2N+1 & 2N+1 \\ \sum_{n=1}^{N} n^3 & 3N^2 & 3N \end{vmatrix}$

$= \begin{vmatrix} \frac{(N+1)N}{2} & 1 & 5 \\ \frac{1}{6}(N+1)N(2N+1) & 2N+1 & 2N+1 \\ \frac{1}{4}(N+1)^2N^2 & 3N^2 & 3N \end{vmatrix}$

[By sum of natural numbers, sum of squares of natural numbers and sum of cubes of natural numbers] Now, taking $\frac{(N+1)N}{12}$ common from C₁ we get

$= \frac{N(N+1)}{12} \begin{vmatrix} 5 & 1 & 5 \\ 2(2N+1) & 2N+1 & 2N+1 \\ 3N & 3N^2 & 3N \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, we get $= \frac{N(N+1)}{12}$

$\begin{matrix} 5 & 1 & 5 \\ 2(2N+1) & 2N+1 & 2N+1 \\ 3N & 3N^2 & 3N \end{matrix}$

= 0
[By properties of determinant-3

27. Answer: (C)

Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Length of latus rectum = 8
⇒ Therefore, $\frac{2b^2}{a} = 8$
b² = 4a……. (1)
conjugate axis = half of the distance between foci
2b = ae…….. (2)
Now, b² = a²(e² – 1)…… (3)
From equation (1) and (3) we get
$\frac{a^2e^2}{4}$ = a2(e2 – 1)
e² = 4e² – 4
e2 = $\frac{4}{3}$
e = $\frac{2}{\sqrt{3}}$

28. Answer: (C)

Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$. It cuts the coordinate axes at $P(a, 0)$ and $Q(0, b)$.
It is given that PQ is bisected at A (3,4).
$\therefore \frac{a+0}{2} = 3$ and $\frac{0+b}{2} = 4$,
$\Rightarrow a = 6, b = 8$
Hence, the equation of the line is $\frac{x}{6} + \frac{y}{8} = 1$ or $4x + 3y = 24$

29. Answer: (A)

Given the equation of hyperbola $9x^2 - 16y^2 + 72xx - 32y - 16 = 0$

$9(x^2+8x)-16(y^2+2y)=16$
$9(x^2+8x+16)-16(y^2+2y+1)=16+144-16$
$3^2(x+4)^2-4^2(y+1)^2=12^2$
$\frac{(x+4)^2}{4^2}-\frac{(y+1)^2}{3^2}=1$
$a^2=16$ and $b^2=9$
a = 4 and b = 3
latus - rectum $\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

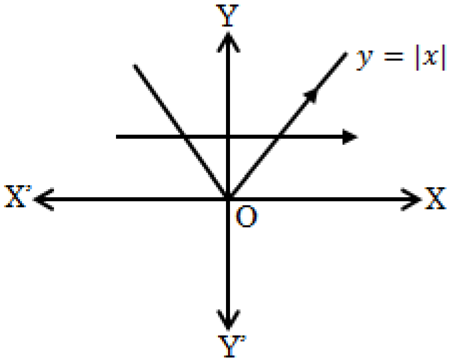
30. **Answer: (C)**
Given $5a=\sec\alpha$
$a=\frac{\sec\alpha}{5}$ ....(1)
Consider, $\frac{5}{a}=\tan\alpha$
$\Rightarrow \frac{1}{a}=\frac{\tan\theta}{5}$..............(2)
From (1) and (2)
$a^2-\frac{1}{a^2}=\left(\frac{\sec\alpha}{5}\right)^2-\left(\frac{\tan\alpha}{5}\right)^2$
$a^2-\frac{1}{a^2}=\frac{1}{25}(\sec^2\alpha-\tan^2\alpha)$
$25\left(a^2-\frac{1}{a^2}\right)=1$

31. **Answer: (D)**
$\Rightarrow g(\alpha)=\int\frac{\sqrt{1-\alpha^2}}{\alpha^4}d\alpha$
Dividing by α in numerator & denominator
$=\int\frac{\sqrt{\frac{1}{\alpha^2}-1}}{\alpha^3}d\alpha$
Let,
$\Rightarrow \frac{1}{\alpha^2}-1=t^2$
$\Rightarrow \frac{-2}{\alpha^3}d\alpha=2tdt$
$\Rightarrow \frac{1}{\alpha^3}d\alpha=-tdt$
$\Rightarrow g(t)=\int t(-tdt)$
$\Rightarrow g(t)=\frac{-t^3}{3}+k$
$\Rightarrow g(\alpha)=\frac{-1}{3}\left(\frac{1}{\alpha^2}-1\right)^{3/2}$
$=\frac{-1\left(\sqrt{1-\alpha^2}\right)^3}{3\alpha^3}$
But,
$\Rightarrow g(\alpha)=f(\alpha)\left[\sqrt{1-\alpha^2}\right]^n+k$ is given
On comparing the two values
$\Rightarrow f(\alpha)=\frac{-1}{3\alpha^3}$ $\quad n=3$
$\Rightarrow [f(\alpha)]^3=\left(\frac{-1}{3\alpha^3}\right)^3$
$\Rightarrow \frac{-1}{27\alpha^3}$

32. **Answer: (A)**
The minimum value of the function exists at $x=c$ on the condition $f'(c)=0$
Now, $f=x^3+2x^2-4x+6$
$f'(x)=3x^2+4x-4$
So, $f'(c)=3c^2+4c-4=$
0 here c is the point where f is minimum.
$\Rightarrow 3c^2+4c-4=0$
$\Rightarrow 3c^2+6c-2c-4=0$
$\Rightarrow (3c-2)(c+2)=0$
$\Rightarrow c=\frac{2}{3},-2$
Thus, the value of c is -2

33. **Answer: (B)**
$f(x)=\frac{x^2-1}{x^2+1}$
For finding the value of x for which f(x) attains minimum value $\frac{df(x)}{dx}=0$
$\Rightarrow \frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}=0$
$\Rightarrow x=0$
$\therefore f(0)=-1$ is the minimum value of $f(x)$

34. **Answer: (D)**
$f: ZN$ and $f(x)=|x|$
When we draw a parallel line to x-axis.
It cuts the curve into more than one point.
Therefore, f(x) =|x| is not one-one.



35. **Answer: (C)**
Given,
Series is in AP, where
First term = a = 17
Total terms = n = 14
Let, common difference = *d*
Also,
Sum = 875
$\because S_n=\frac{n}{2}[2a+(n-1)d]$
$\Rightarrow 875=\frac{14}{2}[2\times 17+(14-1)d]$
$\Rightarrow 125\times 7=7[34+13d]$
$\Rightarrow 125=34+13d$
$\Rightarrow d=\frac{125-34}{13}$
$\Rightarrow d=\frac{91}{13}=7$
Now,
$a_n=a+(n-1)d$
$a_{14}=17+(14-1)7$
$=17+91$
$=108$

36. **Answer: (B)**
For $f(x)$ to be continuous at $=\frac{\pi}{4}$, we must have
$\lim_{x\to\frac{\pi}{4}} f(x)=f\left(\frac{\pi}{4}\right)$
$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot\ 2x}=f\left(\frac{\pi}{4}\right)$
$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\tan\left(\frac{\pi}{4}-x\right)}{\tan\left(\frac{\pi}{2}-2x\right)}=f\left(\frac{\pi}{4}\right)$
$\Rightarrow \frac{1}{2}\lim_{x\to\frac{\pi}{4}}\frac{\left\{\frac{\tan\left(\frac{\pi}{4}-x\right)}{\left(\frac{\pi}{4}-x\right)}\right\}}{\left\{\frac{\tan\ 2\left(\frac{\pi}{4}-x\right)}{2\left(\frac{\pi}{4}-x\right)}\right\}}=f\left(\frac{\pi}{4}\right)$
$\Rightarrow f\left(\frac{\pi}{4}\right)=\frac{1}{2}$

37. **Answer: (A)**
Given, A = (0, 6) and B = (0, -2).
$AB=\sqrt{(0-0)^2+(6+2)^2}=8$

38. **Answer: (D)**
We have,
$\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix}=0$
$\to a(a^2-0)-b(-b^2)+0=0$
$\to a^3+b^3=0$
$\to b^3\left(\frac{a^3}{b^3}+1\right)=0$
$\to \left(\frac{a}{b}\right)^3=1$
$\to (a/b)^3=-1$
a/*b* is one of the cube roots of -1 .

39. **Answer: (B)**
The given function $f(x)=x^4+3x^2+7$
Putting x ⇒ -x

$f(-x) = (-x)^4 + 3(-x)^2 + 7$
$f(-x) = x^4 + 3x^2 + 7$
$f(-x) = f(x)$
Thus function is even.

40. Answer: (A)
Alphabetical order of these letters is B, E, K, R, U.
Total words starting with B = 4! = 24
Total words starting with E = 4! = 24
Total words starting with KB = 3! = 6
Total words starting with KE = 3! = 6
Total words starting with KR = 3! = 6
Next word will be KUBER.
Thus, rank of the word KUBER = 24 + 24 + 18 + 1
= 67.

41. Answer: (B)
$\Rightarrow \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$
$\Rightarrow \int \frac{x^5 - x^4}{x^3 - x^2} dx$
$\Rightarrow \int \frac{x^4(x-1)}{x^2(x-1)} dx [e^{a\log x} = e^{\log x^a} = x^a]$
$\Rightarrow \int x^2 dx$
$\Rightarrow \frac{x^3}{3} + k$

42. Answer: (D)
By simplification we get,
$= \left(\frac{34}{100} \times \frac{11}{100} \times 14\right)$
$= \frac{5236}{10000}$
= 0.5236

43. Answer: (C)
$\frac{42}{23} = 1.82$
$\frac{31}{20} = 1.55$
$\frac{57}{36} = 1.58$
$\frac{82}{51} = 1.60$

44. Answer: (C)
Mean of 100 items $= x_{100} = 50$
Mean of 150 items $= x_{150} = 40$
Standard deviation of 100 items $= \sigma_{100} = 5$
Standard deviation of 150 items $= \sigma_{150} = 6$
$d_1 = 50 - 44 = 6$
$d_2 = 40 - 44 = -4 d_2^{\frac{1}{2}} = 16$
$\sigma_{250} = \frac{\sqrt{n_1(\sigma_{100}^2 + d_1^2) + n_2(\sigma_{150}^2 + d_2^2)}}{n_1 + n_2}$
$= \frac{\sqrt{390}}{5} = \frac{37.28}{5} = 7.456 = 7.5$

45. Answer: (D)
$f'(x) = \begin{cases} -1 & \text{for } 1 \le x \le 2 \\ 3 - 2x & \text{for } x > 2 \end{cases}$
$f(x)$ at $x = 3 f'(3)$
$= 3 - 2(3)$
$= 3 - 6$
$= -3$

46. Answer: (C)
$\int \frac{3}{x^2 + 5x + 6} dx$
$\Rightarrow \int \frac{3}{(x+2)(x+3)} dx$
By partial fraction,
$\Rightarrow \int 3\left(\frac{1}{x+2} - \frac{1}{x+3}\right) dx$
$\Rightarrow 3(\ln(x+2) - \ln(x+3)) + c$
$\Rightarrow 3\ln\left(\frac{x+2}{x+3}\right) + c$

47. Answer: (B)
We have $f(x) = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$
$f(x) = 1 - \frac{1}{2}(2\sin x\cos x)^2 = 1 - \frac{1}{2}\sin^2 2x$
Now, $f'(x) = -\frac{1}{2} \cdot 2 \cdot \sin 2x \cdot \cos 2x \cdot 2 = -\sin 4x$
Since $f(x)$ is increasing
$f'(x) > 0 \Rightarrow -\sin 4x > 0 \Rightarrow \sin 4x < 0$
$\pi < 4x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{4} < x < \frac{3\pi}{8}$

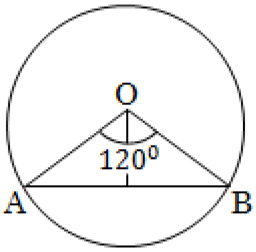
48. Answer: (A)
When the Discriminant of any quadratic equation $(D < 0)$
is less then zero it mean that quadratic curve does not intersect the x – axis anywhere it mean that that equation does not have any real solution.

49. Answer: (B)
we have
$\tan x + \tan 2x + \tan x \cdot \tan 2x$
and $x = 15°$
Now,
$\Rightarrow \tan 15° + \tan 30° + \tan 15° \cdot \tan 30°$
$\tan 15° = \tan(45° - 30°) = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$
$= 2 - \sqrt{3}$
Now,
$\Rightarrow (2 - \sqrt{3}) + \frac{1}{\sqrt{3}} + (2 - \sqrt{3}) \times \frac{1}{\sqrt{3}}$
$= \frac{2\sqrt{3} - 3 + 1 + 2 - \sqrt{3}}{\sqrt{3}} = 1$

50. Answer: (C)
Angle of the sector with the greatest area with fixed perimeter is θ=2 radians
Perimeter = 40= r+r+θr=r+r+2r (θ=2 rad)
r = 10

51. Answer: (B)
Let the equation of the required ellipse is
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$........ (1)
It is given that,
Length of Latus Rectum = $\frac{1}{2}$ major axis
We know that,
Length of Latus Rectum = $\frac{2b^2}{a}$
And Length of Minor axis = 2a
So, according to given condition,
$\frac{2b^2}{a} = \frac{1}{2} \times 2a$
$\Rightarrow \frac{2b^2}{a} = a$
$\Rightarrow 2b^2 = a^2$ .... (2)
$\Rightarrow a = \sqrt{2b^2}$
$\Rightarrow a = b\sqrt{2}$
Now, we have to find the eccentricity
We know that,
Eccentricity, e = $\frac{c}{a}$ .... (3)
Where, $= c^2 = a^2 - b^2$
So, $c^2 = (2b)^2 - b^2$[from (2)]
$\Rightarrow c^2 = b^2 - b^2$
$\Rightarrow c^2 = b^2$
$\Rightarrow c = \sqrt{b^2}$
$\Rightarrow c = b$
Substituting the value of c and a in eq. (3), we get
Eccentricity, e = $\frac{c}{a}$
$= \frac{b}{b\sqrt{2}}$
Therefore, e = $\frac{1}{\sqrt{2}}$

52. Answer: (A)
Given the equation of circle $x^2 + y^2 - 2x - 2y - 2 = 0$
$(x^2 - 2x + 1) + (y^2 - 2y + 1) = 2 + 1 + 1$
$(x - 1)^2 + (y - 1)^2 = 2^2$
So the centre and radius are (1,1) and 2



In $\Delta OPB; \angle OBP = \angle OAP = 30$
$\sin 30 \circ = OP/OA = OP/2$ or $OP = 1$
Let midpoint of chord P(h,k)
So $(h-1)^2 + (k-1)^2 = 1$
$h^2 + k^2 - 2h - 2k + 1 = 0$
Now locus of the mid-point of chord AB
$x^2 + y^2 - 2x - 2y + 1 = 0$

53. **Answer: (B)**
250 numbers between 101 and 350 i.e. $n(S) = 250$
$n(E) = 100^{\text{th}}$ digits of $2 = 299 - 199 = 100$
$P(E) = \frac{n(E)}{n(S)}$
$= \frac{100}{250}$
$= 0.40$

54. **Answer: (C)**
Here, two number are selected from /{1,2,3,4,5,6/}
$\Rightarrow n(S) = 6 \times 5$ (as one by one without replacement)
Favourable events = the minimum of the two number is less than 4
$n(E) = 6 \times 4$
(as for the minimum of the two is less than 4 we can select one from (1,2,3,4) and other from (1,2,3,4,5,6))
Therefore required probability $= \frac{n(E)}{n(S)} = \frac{24}{30} = \frac{4}{5}$

55. **Answer: (C)**
It is given that $T_6 + T_5 = 0$
Hence ${}^n\mathbf{C}_4\mathbf{a}^{n-4}\mathbf{b}^4 - {}^n\mathbf{C}_5\mathbf{a}^{n-5}\mathbf{b}^5 = \mathbf{0}$
${}^nC_4a^{n-4}b^4 = {}^nC_5a^{n-5}b^5$
${}^n\mathbf{C}_4\mathbf{a} = {}^n\mathbf{C}_5\mathbf{b}$
$\frac{\mathbf{n!a}}{(\mathbf{n-4})!\mathbf{4}!} = \frac{\mathbf{n!b}}{(\mathbf{n-5})!\mathbf{5}!}$
$\frac{a}{(n-4)} = \frac{b}{5}$
Therefore
$\frac{a}{b} = \frac{n-4}{5}$

56. **Answer: (A)**
Given that, $A = [a_{lf}]_{2\times 3}$
Highest Rank of $A = min(2,3) = 2$
But, if Rank of A = 2, the system will be consistent.
In order to be inconsistent
Max. Rank of A < min (2,3)
∴ highest possible Rank of A is 1.

57. **Answer: (B)**
The given equation is written as
$\Rightarrow ydx - xdy + (x+y)\sqrt{xy}(xdx + ydy) = 0$
$\Rightarrow \frac{ydx - xdy}{y^2} + \left(\frac{x}{y} + 1\right)\sqrt{\frac{x}{y}}\left(d\left(\frac{x^2+y^2}{2}\right)\right) = 0$
$\Rightarrow d\left(\frac{x^2+y^2}{2}\right) + \frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}+1\right)\sqrt{\frac{x}{y}}} = 0$
$\Rightarrow \frac{x^2+y^2}{2} + 2\tan^{-1}\sqrt{\frac{x}{y}} = c$

58. **Answer: (C)**
We have $\cot\frac{x}{2} - \operatorname{cosec}\frac{x}{2} = \cot x$
$\frac{\cos(x/2)}{\sin(x/2)} - \frac{1}{\sin(x/2)} = \frac{\cos x}{2\sin(x/2)\cos(x/2)}$
$2\cos^2\frac{x}{2} - 2\cos\frac{x}{2} = \cos x$
$1 + \cos x - 2\cos\frac{x}{2} = \cos x$
$\cos\frac{x}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$
$\therefore \frac{x}{2} = 2n\pi \pm \frac{\pi}{3}$
$x = 4n\pi \pm \frac{2\pi}{3}$

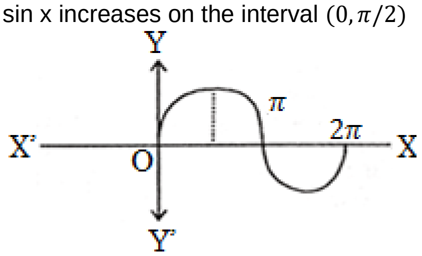
59. **Answer: (C)**
We have,
$y^2 - x^2 + 2x - 1 = 0$
$\Rightarrow y^2 - (x-1)^2 = 0$
$\Rightarrow (y + x - 1)(y - x + 1) = 0$
$\Rightarrow y + x - 1 = 0, y - x + 1 = 0$
Hence, the given equation represents a pair of straight lines.

60. **Answer: (C)**
Given that, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a < b
We know that, $a^2 = b^2(1 - e^2)$

61. **Answer: (A)**
$\Rightarrow \int \frac{x^2+1}{x^2-1}dx$
$= \int \left(1 + \frac{2}{x^2-1}\right)dx$
$= \int 1dx + 2\int \frac{1}{(x-1)(x+1)}dx$
$= x + \int \left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx + c$
$= x + \int \frac{dx}{x-1} - \int \frac{dx}{x+1} + c$
$= x + \ln(x-1) - \ln(x+1) + c$
$= x + \ln\left(\frac{x-1}{x+1}\right) + c$

62. **Answer: (B)**
Let's assume the total no. of marks = x
Percentage calculations = (marks obtained/ total marks) × 100
Then to pass the percentage required
= (1034 / x) × 100 ……..1
Percentage student gets = (940/x) × 100 ……2
Difference of (1) and (2) = 5% = 0.05
$\Rightarrow (1034/x) \times 100 - (940/x) \times 100 = 5$
So, x = 1880

63. **Answer: (D)**
sin x increases on the interval $(0, \pi/2)$



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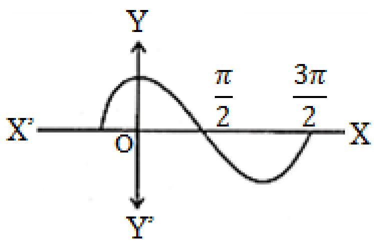


64. **Answer: (B)**
Suresh's age before 10 years x
125 × / 100 = x + 10
125 x = 100x + 1000 ⇒ x = 40
Present age = x + 10 = 50

65. **Answer: (B)**
It is given that points $P(a,b)$ and $Q(b,a)$ lies on the lines $3x + 2y = 13$ and $4x - y = 5$ respectively.
$\therefore 3a + 2b = 13$ and $4b - a = 5$
$\Rightarrow a = 3, b = 2$
Thus, the coordinates of P and Q are (3,2) and (2,3)respectively. So, the equation of line PQ is

$y-2=\frac{3-2}{2-3}(x-3)$

$\Rightarrow x+y=5$

66. Answer: (C)

Given mean of 6 observations is 8.

$\Rightarrow \frac{\sum_{i=1}^{6} X_i}{6}=8$

$\Rightarrow \sum_{i=1}^{6} X_i=48 \ldots\ldots\ldots\ldots (1)$

And Mean of 4 observations is 10.

$\Rightarrow \frac{\sum_{i=1}^{4} X_i}{4}=10$

$\Rightarrow \sum_{i=1}^{4} X_1=40 \ldots\ldots\ldots (2)$

Adding equation (1) and (2)

$\sum_{i=1}^{6} X_i+\sum_{i=1}^{4} X_i=48+40=88$

$\Rightarrow \sum_{i=1}^{10} X_i=88$

Hence . Mean of 10 observations $=\frac{\sum_{i=1}^{10} X_i}{10}=\frac{88}{10}=8.8$

67. Answer: (B)

$f'(0-)=\lim_{t\to 0^-}\frac{|t|\left(3e^{\frac{1}{|t|}}+4\right)-0}{\frac{\frac{1}{2-e^{\frac{1}{|t|}}}}{t-0}}$

$=\frac{-t\left(3e^{-\frac{1}{t}}+4\right)-0}{\lim_{t\to 0}\frac{2-e^{-\frac{1}{t}}}{t-0}}=\frac{-(3e^{-x}+4)}{2-e^{-\infty}}=-2$

$f'(0+)=\lim_{t\to 0^+}\frac{|t|\left(3e^{\frac{1}{|t|}}+4\right)-0}{\frac{\frac{1}{2-e^{|y|}}}{t-0}}$

$=\lim_{t\to 0}\frac{t\left(3e^{\frac{1}{t}}+4\right)-0}{\frac{\frac{1}{2-e^{\frac{1}{t}}}}{t-0}}=\frac{\frac{3+\frac{4}{e^{\frac{1}{t}}}}{2}}{\frac{1}{\frac{1}{e^{6}}-1}}=\frac{3}{-1}=-3$

$\Rightarrow f(0-)\neq f'(0+)$

Hence, f is not differentiable at t = 0 and continuous at t = 0.

68. Answer: (B)

We know that the equation of a plane whose intercepts on the coordinate axes are *a*, *b* and *c* respectively, is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Here, $a=-4, b=2$ and $c=3$.

So, the equation of the required plane is

$\frac{x}{-4}+\frac{y}{2}+\frac{z}{3}=1$

$\Rightarrow -3x+6y+4z=12$

69. Answer: (A)

We have,

$|z|=k$

$\Rightarrow |z|^2=k^2$

$\Rightarrow z\bar{z}=k^2[\because z^-=z^2]$

$\Rightarrow \bar{z}=\frac{k^2}{z}$

Now, 2Re $(\omega)=\omega+\bar{\omega}$ and $\omega=\frac{z-k}{z+k}$

$\Rightarrow$ Re $(\omega)=\frac{1}{2}\left\{\frac{z-k}{z+k}+\frac{\bar{z}-k}{\bar{z}+k}\right\}$

$\Rightarrow$ Re $(\omega)=\frac{1}{2}\left\{\frac{z-k}{z+k}-\frac{z-k}{z+k}\right\}=0\left[\because \bar{z}=\frac{k^2}{z}\right]$

70. Answer: (D)

The ways of selecting three distinct numbers are chosen randomly from the first 100 natural numbers is $^{100}C_3$

We know that a number will be divisible by both 2 and 3, if it divisible by their l.c.m. i.e. 6

There are 16 numbers, in first natural numbers, which are divisible by 6. Therefore, number of ways of selecting 3 numbers such that all of them are divisible by both 2 and 3 is .

Hence,

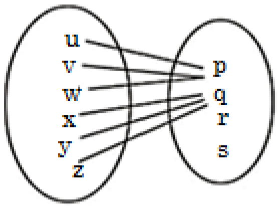
Required probability $=\frac{16_{C_3}}{100_{C_3}}=\frac{16\times15\times14}{100\times99\times98}=\frac{4}{1155}$

71. Answer: (C)

Given that, A = {u, v, x, y, z} ; B = {p, q, r, s}

As we know, a mapping f:x → is said to be a function, if each element in the set x has its image in set y.

It is also possible that these are few elements in set y which are not the image of any element in set x. Every element in set x should have one and only one image.



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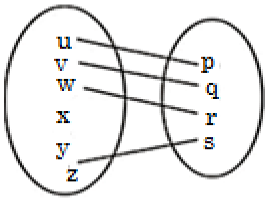

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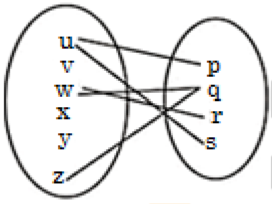

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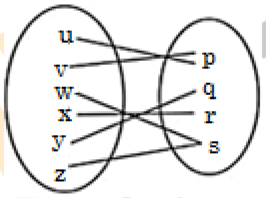


4.

72. Answer: (A)

SI of 2 years =Rs.4000

Difference between S and C at the end of 2 years = 4100 – 4000 = Rs.100

Rate of interest $=\frac{100}{2000}\times 100=5\%$

$\frac{P\times5\times3}{100}=6000$

$P=Rs.40000$

73. Answer: (B)

Given, series is 3, 6, 9, 12………………,198 in AP,

First term = a = 3

Common difference = d = 3

Last term = $l=a_n=198$

Let, no. of terms = n

$\because a_n=a+(n-1)d$

$\Rightarrow 198=3+(n-1)3$

$\Rightarrow n-1=\frac{198-3}{3}$

$\Rightarrow n-1=\frac{195}{3}=65$

$\Rightarrow n=66$

Now, for middle term $n'=\frac{n}{2}=\frac{66}{2}=33$

$\Rightarrow a_{33}=a+(n'-1)d$

$\Rightarrow a_{33}=3+(33-1)3$

$\Rightarrow a_{33}=3+32\times3$

$\Rightarrow a_{33}=3+96=99$

Hence, the middle term will be 99

74. Answer: (C)

we have

$LHS=\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$

$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta}-\frac{1-\tan\theta}{1+\tan\theta}$

$\Rightarrow \frac{(1+\tan\theta)^2-(1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}$

$\Rightarrow \frac{4\tan\theta}{1-\tan^2\theta}$

$\Rightarrow 2\tan 2\theta$

75. Answer: (A)

We have,

$\lim\limits_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}$

$=\lim\limits_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}\times\frac{2+\sqrt{f(x)}}{2+\sqrt{f(x)}}\times\frac{2+\sqrt{x}}{2+\sqrt{x}}$

$=\lim\limits_{x\to 4}\frac{4-f(x)}{4-x}\times\frac{2+\sqrt{x}}{2+\sqrt{f(x)}}$

$=\lim\limits_{x\to 4}\frac{f(x)-4}{x-4}\times\lim\limits_{x\to 4}\frac{2+\sqrt{x}}{2+\sqrt{f(x)}}$

Using L' Hospital rule, $\frac{0}{0}$ form

$=f'(4)\times\frac{2+2}{2+\sqrt{f(4)}}$

[$\because f(x)$ is differentiableat $x=4$
$\therefore$ It is continuous at $x=4$.
Hence, $\lim\limits_{x\to 4}f(x)=f(4)$]

$=1\times\frac{4}{2+2}=1$

76. **Answer: (C)**
B × B = {2, 3} × {2, 3}
= [{2, 2}, {2, 3},{3, 2},{3, 3}]

77. **Answer: (D)**
Given, *a, 4, b* are in AP
∴ by middle term concept, we get

$\frac{a+b}{2}=4$

$\Rightarrow a+b=8\to(i)$

Also, *4, a, b* are in GP
Similarly, by middle term concept, we get

$a^2=4b$

$\Rightarrow b=\frac{a^2}{4}\to(ii)$

Put eq. (ii) in (i),

$a+\frac{a^2}{4}=8$

$\Rightarrow a^2+4a-32=0$

$\Rightarrow a^2+8a-4a-32=0$

$\Rightarrow a(a+8)-4(a+8)=0$

$\Rightarrow (a-4)(a+8)=0$

$\Rightarrow a=-8,4$

Now, put value of *a* in eq. (i)
when $a=-8$, when $a=4$
$-8+b=8$ $4+b=8$
$\Rightarrow b=16$ $\Rightarrow b=4$
Also given, $a\neq b$
So,
$a=-8$
$b=16$

78. **Answer: (A)**
We have,
We know that the necessary and sufficient condition for $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of straight lines is

$abc+2fgh-af^2-bg^2-ch^2=0$

This will be represent a pair of straight lines, if

$6k+(-7)(-9/2)(7/2)-2\times\frac{49}{4}-3\times\frac{81}{4}-k\times\frac{49}{4}=0$

$\Rightarrow 6k+\frac{9\times 49}{4}-\frac{49}{2}-\frac{243}{4}-\frac{49k}{4}=0$

$\Rightarrow k=4$

79. **Answer: (B)**
Statement 1:
Students, who had taken only one subject
= 2 + 5 + 4 = 11
Students, who had taken only two subjects
= 6 + 2 + 1= 9
1 ≠ 9
Statement 2:
Students who had taken at-least two subject
= 1 + 2 + 6 + 3 = 12
Students who had taken all three subjects
= 4 × 3 = 12

80. **Answer: (C)**
Only two subjects = 6 +2 + 1 = 9

81. **Answer: (A)**
Only Physics = 12 - (1 + 3 + 6) = 2

82. **Answer: (D)**
$\log a,\log b,\log c$ are in A.P.

$\Rightarrow 2\log b=\log a+\log c$

$\Rightarrow \log b^2=\log(ac)$

$\Rightarrow b^2=ac\Rightarrow a,b,c$ are in G.P.

$\log a-\log 2b,\log 2b-\log 3c,\log 3c-\log a$ are in A.P.

$\Rightarrow 2(\log 2b-\log 3c)=(\log a-\log 2b)+(\log 3c-\log a)$

$\Rightarrow 3\log 2b=3\log 3c\Rightarrow 2b=3c$

Now, $b^2=ac\Rightarrow b^2=a\cdot\frac{2b}{3}\Rightarrow b=\frac{2a}{3},c=\frac{4a}{9}$

i.e., $a=a,b=\frac{2a}{3},c=\frac{4a}{9}$

$\Rightarrow a:b:c=1:\frac{2}{3}:\frac{4}{9}=9:6:4$

Since, sum of any two is greater than the 3rd, a, b, c form a triangle.

83. **Answer: (A)**
The given data is : 6,7,4,8,9,10,3,5
Here, the numbers of observations are 8, which is even.
Arranging the data in ascending order, we obtain
3,4,5,6,7,8,9,10

Median, M = $\frac{\left(\frac{8}{2}\right)^{th}\text{ observation}+\left(\frac{8}{2}+1\right)^{th}\text{ observation}}{2}$

$M=\frac{(4)^{th}\text{ observation}+(5)^{th}\text{ observation}}{2}=\frac{6+7}{2}=\frac{13}{2}=6.5$

The deviations of the respective observations from the median, i.e. x_i-M, are

$-3.5,-2.5,-1.5,-0.5,0.5,1.5,2.5,3.5$

The absolute values of the deviations, $|x_i-M|$ are
3.5,2.5.1.5,0.5,0.5,1.5,2,5.3.5
The required mean devaition about the median is

Mean Deviation $=\frac{\sum_{i=1}^{8}(X_1-M)}{8}=\frac{3.5+2.5+1.5+0.5+0.5+1.5+2.5+3.5}{8}=\frac{16}{8}=2$

84. **Answer: (B)**
We have,

$f(x+y)=f(x)+\lambda xy+3x^2y^2$

Putting $x=3$ and $y=2$, we get

$f(5)=f(3)+6\lambda+108$

$\Rightarrow 52=4+6\lambda+108$

$\Rightarrow \lambda=-10$

$\therefore f(x+y)=f(x)-10xy+3x^2y^2$

$\Rightarrow\frac{f(x+y)-f(x)}{y}=-10x+3x^2y$

$\Rightarrow\lim\limits_{y\to 0}\frac{f(x+y)-f(x)}{y}=\lim\limits_{y\to 0}-10x+3x^2y$

$\Rightarrow f'(x)=-10x$

85. **Answer: (A)**

$\int\frac{4}{\sqrt{-x^2+4x}}dx$

$\Rightarrow\int\frac{4}{\sqrt{-(x^2-4x+2^2)+2^2}}dx$

$\Rightarrow\int\frac{4}{\sqrt{2^2-(x-2)^2}}dx$

$\Rightarrow 4\sin^{-1}\frac{(x-2)}{2}+c$

86. **Answer: (B)**
Given,
a,b,c and u,v,w are the vertices of two triangles such that $c=(1-r)a+rb$ and $w=(1-r)u+rv$

Consider, $=\begin{vmatrix}a & u & 1\\ b & v & 1\\ c & w & 1\end{vmatrix}$

Applying $R_3\to R_3-[(1-r)R_1-rR_2]$,we get

$=\begin{vmatrix}a & u & 1\\ b & v & 1\\ c-(1-r)a-rb & w-(1-r)u-rv & 1-(1-r)-r\end{vmatrix}$

$=\begin{vmatrix}a & u & 1\\ b & v & 1\\ c & w & 1\end{vmatrix}$ [Using (i)]

Using properties of determinant- 5 = 0
Therefore, two triangles are similar.

87. **Answer: (D)**

$\lim_{x\to\frac{x}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{(\pi-2x)^2}$

$\lim_{x\to\frac{x}{2}} \frac{-\cos x}{(x-2x)(-2)}$

$\lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{4(\pi-2x)} = \lim_{x\to\frac{\pi}{2}} \frac{\sin x}{8}$

$= \frac{1}{8}\cdot\sin\frac{\pi^2}{2} = \frac{1}{8}\times 1 = \frac{1}{8}$

88. **Answer: (C)**

We have ,

$f(x) = xe^{x(1-x)}$

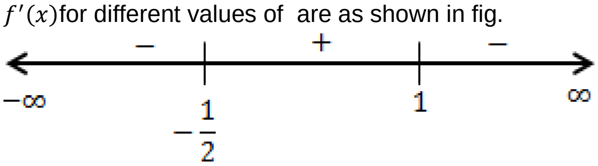
Differentiating wrt x,

$\Rightarrow \quad f'(x) = e^{x(1-x)} + x(1-2x)e^{x(1-x)} \left[\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx} \& \frac{d}{dx}e^x = e^x\frac{dx}{dx}\right]$

$\Rightarrow f'(x) = (1+x-x^2)e^{x(1-x)}$

$\Rightarrow f'(x) = -(x-1)(2x+1)e^{x(1-x)}$

Since $xe^{x(1-x)} > 0$ for all . Therefore, signs of $f'(x)$for different values of are as shown in fig.



Clearly, $f(x)$ is increasing on $[-1/2,1]$ and decreasing on $[-\infty,-1/2] \cup [1,\infty)$.

89. **Answer: (D)**

The given function is $f(x) = \sin x - \cos x$

$f'(x) = \cos x + \sin x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right) = \sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$

The function is decreasing when, $f'(x) < 0 \Rightarrow \sqrt{2}\sin\left(\frac{\pi}{4}+x\right) < 0$

We know that $\sin x$ is decreasing in $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ in the range $0 \le x \le 2\pi$

So, $\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{3\pi}{2} \Rightarrow \frac{\pi}{4} < x < \frac{5\pi}{4}$

90. **Answer: (D)**

450° = 90° means, after having completed one circle of 360°, the position of the angle will be at 90°

tan450° = tan 90°

91. **Answer: (C)**

The given expression

$$= \left[\sqrt{9^{x-1}+7} + \frac{1}{(3^{x-1}+1)^{\frac{1}{5}}}\right]^7$$

Given, $T_6 = \mathbf{84}$

$\Rightarrow {}^7C_5\left(\sqrt{9^{x-1}+7}\right)^{7-5}\left(\frac{1}{(3^{x-1}+1)^{\frac{1}{5}}}\right)^5 = 84$

$\Rightarrow {}^7C_5(9^{x-1}+7)\cdot\frac{1}{(3^{x-1}+1)} = 84$

$\Rightarrow 9^{x-1}+7 = 4(3^{x-1}+1)$

$\Rightarrow 3^{2x} - 12.3^x + 27 = 0$

$\Rightarrow (3^x-3)(3^x-9) = 0$

$\Rightarrow 3^x = 3,9$

$\Rightarrow \mathbf{x = 1, 2}$

92. **Answer: (A)**

We have $\tan\alpha\cdot\tan(2n-1)\alpha$

Given : $n = \frac{\pi}{4\alpha}$

$\tan\alpha\cdot\tan\left(2\cdot\frac{\pi}{4\alpha}-1\right)\alpha$

$= \tan\alpha\cdot\tan\left(\frac{\pi}{2\alpha}-1\right)\alpha$

$= \tan\alpha\cdot\tan\left(\frac{\pi}{2}-\alpha\right)$

$= \tan\alpha\cdot\cot\alpha \left[\because \tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta\right]$

$= 1$

93. **Answer: (A)**

We have,

$f(x) = \log_e|x-1| = \begin{cases}\log(x-1), & \text{if } x>1\\ \log(1-x), & \text{if } x<1\end{cases}$

$\therefore f'(x) = \begin{cases}\frac{1}{x-1}, & \text{if } x>1\\ \frac{1}{x-1}, & \text{if } x<1\end{cases} \left[\because \frac{d\log_e x}{dx} = \frac{1}{x}\right]$

$\Rightarrow f'(x) = \frac{1}{x-1}$ for all $x \ne 1$

$\Rightarrow f'\left(\frac{1}{2}\right) = -2$

94. **Answer: (C)**

$\frac{1+\sin(A)}{1-\sin(A)} - \frac{1-\sin(A)}{1+\sin(A)}$

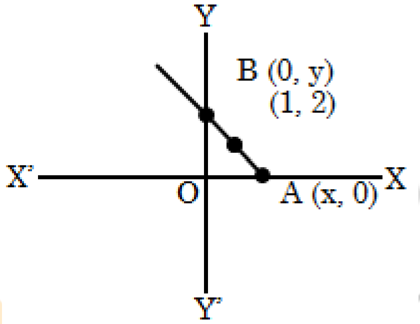
$= \frac{(1+\sin(A))^2-(1-\sin(A))^2}{(1-\sin(A))(1+\sin(A))}$

$= \frac{4\times\sin(A)\times 1}{(1^2)-\sin(A)^2}$

$= \frac{4\sin(A)}{\cos A^2}$

$= 4\tan A\sec A$

95. **Answer: (C)**



$\frac{0+x}{2} = 1; \frac{0+y}{2} = 2$

$X = 2, y = 4$

Equation of line passing through $(2,0)$ and $(0,4)$ $y - 0 = \frac{4-0}{0-2}(x-2)$

$y = -2x+4$

$2x + y = 4$

96. **Answer: (B)**

$=> ydx + xdy = -x^2ydy$

$\Rightarrow \frac{ydx+xdy}{(xy)^2} = \frac{-dy}{y}$

$\Rightarrow \frac{\frac{d}{dx}(xy)}{(xy)^2} = -\frac{dy}{y}$

$-\frac{1}{xy} = -\log y + k$

$\Rightarrow -\frac{1}{xy} + \log y = c$

97. **Answer: (A)**

$\mathbf{I} = \int \frac{\mathbf{dx}}{\cos^3 x\sqrt{2\sin 2x}}$

$\Rightarrow \mathbf{I} = \int \frac{\sec^3 x}{2\sqrt{\sin x\cos x}}dx$

Multiplying numerator and and denominator of I by secx, we get

$I = \int \frac{\sec^4 x}{2\sqrt{\tan x}}dx$

Now assuming, $\tan x = t^2 \Rightarrow \sec^2 x dx = 2tdt$ we get,

$\mathbf{I} = \int \frac{\mathbf{1+t^4}}{\mathbf{2t}}(\mathbf{2t})\mathbf{dt}$

$\Rightarrow \mathbf{I} = \int(\mathbf{1+t^4})\mathbf{dt}$

$\Rightarrow \mathbf{I} = \mathbf{t} + \frac{\mathbf{t^5}}{\mathbf{5}} + \mathbf{K}$

Re-Substituting t $=\sqrt{\tan x}$, we get

$I = \sqrt{\tan x} + \frac{1}{5}\cdot\sqrt{\tan x}^5 + K$

Comparing it with the given expression

$I = \sqrt{\tan x} + \frac{1}{5}\cdot\sqrt{\tan x}^5 + K$

$= (\tan x)^A + C(\tan x)^B + K$

$\Rightarrow \mathbf{A} = \frac{1}{2}, \mathbf{B} = \frac{5}{2}, \mathbf{C} = \frac{1}{5}$

$\mathbf{A+B+C} = \frac{1}{2} + \frac{5}{2} + \frac{1}{5} = \frac{16}{5}$

98. **Answer: (C)**

Let,

$\Rightarrow f(y) = \int y^5 e^{-y^2}dy$

Let
$\Rightarrow y^2 = t$
$\Rightarrow ydy = \frac{1}{2}dt$
$\Rightarrow f(t) = \int t^2 e^{-t}\left(\frac{1}{2}dt\right)$
Using integration by parts
$\Rightarrow f(t) = \frac{1}{2}\left[\{t^2\int e^{-t}dt\} - \int\left\{\frac{dt^2}{dt}\cdot\int e^{-t}dt\right\}dt\right]$
$= \frac{1}{2}\left[t^2e^{-t} + \int 2te^{-t}dt\right]$
$= \frac{-1}{2}t^2e^{-t} + \int te^{-t}dt$ (again using integration by parts
$= \frac{-t^2e^t}{2} + [\{t\int e^{-t}dt\} - \{\int e^{-t}dt\}dt]$
$= \frac{-t^2e^t}{2} + [-te^{-t} - e^{-t}]$
$= \frac{-e^{-t}}{2}[t^2 + 2t + 2]$
$\Rightarrow f(t) = \frac{-1}{2}e^{-t}[t^2 + 2t + 2]$
$\Rightarrow f(y) = \frac{-1}{2}e^{-y}[y^2 + 2y + 2]$
On comparing with given equation
$\Rightarrow g(y) = \frac{-1}{2}[y^4 + 2y^2 + 2]$
$\Rightarrow g(+1) = \frac{-1}{2}[1 + 2 + 2]$
$\Rightarrow g(1) = \frac{-5}{2}$

99. Answer: (A)
Given determinant,
2a(bc – 4a²) + b(2ac –b²) + c (2ab – c²) = 0
$6abc - 8a^3 - b^3 - c^3 = 0$
$(2a + b + c)[(2a - b)^2 + (b - c)^2 + (c - 2a)^2] = 0$
$2a + b + c = 0$ ($\because b \neq c$)
Let $f(x) = 8ax^3 + 2bx^2 + cx$
$f(0) = 0$
$f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{2} = \frac{2a+b+c}{2} = 0$
So, f(x) satisfies the Rolle's theorem and hence,
f'(x) = 0 has at least one root in $\left[0, \frac{1}{2}\right]$

100. Answer: (B)
We have,
$f(x) = (x-1)^2 + 3$
$\Rightarrow f'(x) = 2(x-1) < 0$ for all $x \in [-3,1]$
So, $f(x)$ is decreasing $[-3,1]$
$\Rightarrow m = f(1)$ and $M = f(-3)$
$\Rightarrow m = 3$ and $M = 19$
$\therefore (m, M) = (3,19)$

101. Answer: (D)
$10_{C_4}\left(\frac{1}{2\sqrt{x}}\right)^6\left(\frac{1}{2}\right)^4 = 105$
$x^3 = \frac{10_{C_4}}{105\times 2^{10}}$
$x = \frac{1}{8}$

102. Answer: (C)
$\left(x^3 - \frac{1}{x^2}\right)^n$
General term, $T_{r+1} = {}^nC_r(x^3)^{n-r}\cdot\left(\frac{1}{x^2}\right)^r$
$= {}^nC_r, 3^{(3n-3r)}\cdot(-1)^r\cdot r^{-2r}$
$= {}^nC_r\cdot(-1)^r\cdot x^{(3n-5r)}$
For the coefficient x^5
Put $3n - 5r = 5$
$5r = 3n - 5$
$\therefore r = \frac{3n}{5} - 1$
$\therefore$ coefficient of $x^5 = {}^nC\left(\frac{3n}{5} - 1\right)^{(-1)}\left(\frac{3n}{5} - 1\right)$
For the coefficient of x^{10}
Put $3n - 5r = 10$
$5r = 3n - 10$
$\therefore r = \frac{3n}{5} - 2$
$\therefore$ coefficent of $x^{10} = {}^nC_{\left(\frac{3n}{5}-2\right)}(-1)^{\left(\frac{3n}{5}-2\right)}$
The sum of the coefficient of x^5 and $x^{10} = 0$
$\Rightarrow {}^nC_{\left(\frac{3n}{5}-1\right)}(-1)^{\left(\frac{3n}{5}-1\right)} + {}^nC_{\left(\frac{3n}{5}-2\right)}(-1)^{\left(\frac{3n}{5}-2\right)} = 0$
$\Rightarrow (-1)^{\frac{3n}{5}}\left[nC_{\left(\frac{3n}{5}-1\right)}\cdot(-1)^{-1} + {}^nC_{\left(\frac{3n}{5}-2\right)}(-1)^{(-2)}\right] = 0$
$\Rightarrow -{}^nC_{\left(\frac{3n}{5}-1\right)} + {}^nC\left(\frac{3n}{5} - 2\right) = 0$
From equation (ii) ${}^nC\left(\frac{3n}{5} - 2\right) = {}^nC_{\left(\frac{3n}{5}-1\right)}$
$\Rightarrow$ n $= \left(\frac{3n}{5} - 2\right) + \left(\frac{3n}{5} - 1\right)\left[\because {}^nC_x = {}^nC_y \Rightarrow n = x + y\right]$
$\Rightarrow n = \frac{6n}{5} - 3 \Rightarrow \frac{6n}{5} - n = 3$
$\Rightarrow \frac{n}{5} = 3$
$\therefore n = 15$

103. Answer: (C)
Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it. The tens place can now be filled by any of the remaining 5 digits (2,3,6,7,9). So, there are 5 ways of filling the tens place. The hundreds place can now be filled by any of the remaining 4 digits.
So, there are 4 ways of filling it.
∴ Required number of numbers = (1 × 5 × 4) = 20

104. Answer: (B)
Profit% reduced $= 6 - 2.4 = 3.6\%$
∴ Required $CP = \frac{180}{3.6}\times 100$
$= Rs. 5000$

105. Answer: (A)
Note that the Discriminant is negative:
D = b² − 4ac = 2² − 4×5×1= −16
hence non-real roots

106. Answer: (A)
Let $r_1, r_2 \& r_3$ are the exradii of the triangle …i
$r_1 = \frac{\Delta}{(3-a)}, r_2 = \frac{\Delta}{(3-b)}, r_3 = \frac{\Delta}{(3-c)}$ ...
where Δ is the area of triangle and $s = \frac{a+b+c}{2}$
The harmonic mean of the three exradii are
$HM = \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$ …ii
Putting the value of r1,r2,r3 from (1) to (2)
$HM = \frac{3\Delta}{(s-a+s-b+s-c)}$
$= \frac{3\Delta}{s}$
$= 3r$ $\left[r = \frac{\Delta}{s}\right]$

107. Answer: (C)
Let $I = \int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\{1-(\sin x-\cos x)^2\}}dx$
$\therefore I = \int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{25-16(\sin x-\cos x)^2}dx$
Substitute, 4(sinx − cosx) = t
$\Longrightarrow 4(\cos x + \sin x)dx = dt$
$\frac{1}{4}\int_{-4}^{0}\frac{dt}{25-t^2}$
$I = \frac{1}{4}\cdot\frac{1}{2(5)}\left(\log\left|\frac{5-t}{5+t}\right|\right)^0$
$= \frac{1}{40}\left[\log\left|\frac{5-0}{5+0}\right| - \log\left|\frac{5+4}{5-4}\right|\right]$
$= \frac{1}{40}\left[\log 1 - \log\left(\frac{1}{9}\right)\right]$, (where $\log 1 = 0$)
$= \frac{1}{40}[\log 9] = \frac{2}{40}\log 3 = \frac{1}{20}(\log 3)$

108. Answer: (A)
Given $f(x) = \frac{3x+2}{5x-3}$, assuming $f(x) = y$ then $f^{-1}(y) = x$
So $f(x) = y = \frac{3x+2}{5x-3}$
$5xy - 3y = 3x + 2$
$x(5y - 3) = 2 + 3y$
$x = \frac{3y+2}{5y-3}$
If we put y = x then it becomes $f(x)$
Thus $f^{-1}(x) = f(x)$

109. **Answer: (D)**
We have $\sin 9\theta = \sin \theta$
$\therefore 9\theta = n\pi + (-1)^n\theta$
If *n* is even,
$\Rightarrow 9\theta = n\pi + \theta$
$\Rightarrow \theta = \frac{n\pi}{8}$
If *n* is odd,
$\Rightarrow 9\theta = n\pi - \theta$
$\Rightarrow \theta = n\frac{\pi}{10}$

110. **Answer: (B)**
Let the cost price of goods = 100
SP of goods = 125% of 100 = 125
MP of goods = 125 × 100/90 =1250/9
Difference of MP and CP
= 1250/9 - 100 = (1250 - 100)/9 = 350/9
Difference percentage = (350/9)/100 × 100
⇒ 350/9 = 39% (approx.)

111. **Answer: (A)**
$3, \sqrt{3}, 1, \frac{1}{\sqrt{3}}, \dots \dots, \infty$
This is a Geometric Progression with $a = 3, r = \frac{1}{\sqrt{3}}$
$S_\infty = \frac{a}{1-r} = \frac{3}{1-\frac{1}{\sqrt{3}}}$
$\frac{3\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{3\sqrt{3(\sqrt{3}+1)}}{2}$

112. **Answer: (B)**
$P = 16\ (4^2)$
$Q = 9\ (3^2)$
$R = 4\ (2^2)$
Age after 5 years $P = 16 + 5 = 21$ years
$Q = 9 + 5 = 14$ years
$R = 4 + 5 = 9$ years
Total $= 21 + 14 + 9 = 44$
Age of P after 3 years $= 16 + 3 = 19$

113. **Answer: (C)**
At end of 1st year $8000 + 1600 = 9600$
Amount $9600 - 3000 = 6600$
At the end of 2nd year $6600 + 1320 = 7920$
Amount $7920 - 3000 = 4920$
Amount to be paid at the end of third year $= 4920 + 984 = 5904$

114. **Answer: (D)**
A ×B ={1, 3, 5} × {2,3} = [{1, 2},{1, 3},{3, 2},{3, 3},{5, 2},{5, 3}]

115. **Answer: (C)**
$(a + b + c)^2 \geq 0$
$3 + 2(a \cdot b + b \cdot \hat{c} + \vec{c} \cdot \vec{a}) \geq 0$
$3 + 6\cos\theta \geq 0$
$\cos\theta \geq -\frac{1}{2}$
$\theta = \frac{2\pi}{3}$

116. **Answer: (B)**
We have $3\tan^2\theta - 2\sin\theta = 0$
$3\frac{\sin^2\theta}{\cos^2\theta} - 2\sin\theta = 0$
$3\sin^2\theta - 2\sin\theta(1 - \sin^2\theta) = 0$
$\sin\theta(2\sin^2\theta + 3\sin\theta - 2) = 0$
$\sin\theta(2\sin\theta - 1)(\sin\theta + 2) = 0$
$\sin\theta = 0$ or $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$ and $\sin\theta \neq -2$
$\therefore \theta = n\pi$ or $n\pi + (-1)^n\frac{\pi}{6}$

117. **Answer: (B)**
Let
$\vec{\alpha} = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{+|\vec{b}|} + \frac{\vec{c}}{|\vec{a}|}$
Since $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, if $\vec{\alpha}$ makes angles $\theta, \phi\psi$ with $\vec{a}, \vec{b}$ and $\vec{c}$ respectively, then
$\vec{\alpha} \cdot \vec{a} = \frac{\vec{a} \cdot \vec{a}}{|\vec{a}|}$
$|\vec{\alpha}| \cdot |\vec{a}|\cos\theta = \vec{a}\ |$
$|\vec{\alpha}| \cdot |\vec{a}|\cos\theta = |\vec{a}|$
$\cos\theta = \frac{1}{|\vec{\alpha}|}$
Similarly,
$\cos\phi = \frac{1}{|\vec{\alpha}|}, \cos\psi = \frac{1}{|\vec{\alpha}|}$
$\theta = \phi = \psi$

118. **Answer: (A)**
We have,
$\frac{1}{2}, \frac{1}{3}, n$ are direction cosines of a line.
$\therefore l^2 + m^2 + n^2 = 1$
$\Rightarrow \frac{1}{4} + \frac{1}{9} + n^2 = 1$
$\Rightarrow n^2 = \frac{23}{36}$
$\Rightarrow n = \frac{\sqrt{23}}{6}$

119. **Answer: (C)**
$\lim_{x\to 0}\frac{(1+x)^n - 1}{x}$
$= \lim_{x\to 0}\frac{n_{C_0} + n_{C_1} + n_{C_2}x^2 + \dots + C_n x^n - 1}{x}$
$\lim_{x\to 0}\frac{x(n_{C_1} + n_{C_2}x + \dots + n_{C_1}x^n - 1)}{x}$
$\lim_{x\to 0} n_{C_1} + n_{C_2}x + \dots + n_{C_1}x^n - 1$
Put $x = 0 \Rightarrow n_{C_1} = n$

120. **Answer: (C)**
Equation of line passing through (0,1) and slope =2
$y - 1 = 2(x - 0)$
$y - 2x + 1$
Let line meets at (x, 0)
$0 = 2x_1 + 1 \Rightarrow x_1 = -\frac{1}{2}$
Tangent to the curve at (0,1) meets the $\left(-\frac{1}{2}, 0\right)$